A Problem Booklet for IIM-CAT 2008

School of Management
Quantitative Aptitude (Volume II)

A Sample Booklet(not for sale)

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Problem Set

101)Quadrilateral $ABCD$ is inscribed in a circle with diameter $AD = 4$, if side AB and BC each have length 1, then find CD?

a) 5/2 b) 3 c) 7/2 d) 4 e) none of these

102)Given that a,b and c are roots of the equation $x^3 - ax^2 + bx - c = 0$. find the number of ordered triplets (a, b, c)

a)0 b) 1 c) 2 d) 3 e) none of these

103)Find the least multiple of 17 which when divided by $2, 3, 4..16$ leaves a remainder of 1 in each case
Solve without option :)

104)Find the number of ordered pairs (x, y) of integers such that
$55x^2 + 2xy + y^2 = 2007$

a) 2 b) 4 c) 6 d) 8 e) 12

105)Let $X = 1, 2, 3...k$ and let S be any non-empty collection of subsets of X. Define S' to be collection of all subsets of X that are subsets of an odd number of elements in S. Then

a) $S' = S$ b) $(S')' = S$ c) $S' = X$ d) $(S')' = X$
e) none of the above

106)The diagonals AC and CE of a regular hexagon $ABCDEF$ are divided by the internal points M and N such that $\frac{AM}{AC} = \frac{CN}{CE} = r$. Determine r if B, M and N are collinear.

a) 1 b) 1/3 c) 2/3 d) 1/2 e) none of these

107)Abhishek fills numbers from '1' to '5' in a grid with 5 columns & 100 rows such that:

1)The number entered in any cell represents its column number from the left.
2)No two non-empty rows are identical.

Find the maximum number of cells that he can fill in the grid

1) 31 2)6 3)77 4)75 5)80.

108) $A + B + C + D = D + E + F + G = G + H + I = 17$ where each letter represent a number from $1\ to\ 9$.

Find out number of ordered pairs (D, G) if letter $A = 4$.

a) 0 b) 1 c)2 d) 3 e) none of these

109)The sequence $1, 3, 4, 9, 10, 12.....$ includes all numbers that are a sum of one or more distinct powers of 3. Then the 50_{th} term of the sequence is

a. 252 b. 283 c. 327 d. 360 e) none of these

110) Given that $g(h(x)) = 2x^2 + 3x$ and $h(g(x)) = x^2 + 4x - 4$ for all real x. WHich of the following could be the value of g(-4)?
a)1 b) -1 c) 2 d) -2 e) -3

111) If $a, x, b\ and\ y$ are real numbers and $ax + by = 4$ and $ax^2 + by^2 = 2$ and $ax^3 + by^3 = -3$ then find $(2x - 1)(2y - 1)$

a)4 b) 3 c) 5 d) -3 e) cannot be determined.

112)$K_1, K_2, K_3...K_{30}$ are thirty toffees. A child places these toffees on a circle, such that there are exactly n (n is a positive integer) toffees placed between K_i and K_{i+1} and no two toffees overlap each other.

a)4 b) 5 c) 9 d) 12 e) 13

113) For the n found in question 112, which of the two toffees are adjacently placed on the circle? (All other conditions remaining same)
a) $K_{11}\ and\ K_{13}$ b) $K_6\ and\ K_{23}$ c) $K_2\ and\ K_{10}$ d) $K_{11}\ and\ K_{18}$
e) $K_{20}\ and\ K_{28}$

114) A is a 4-digit positive integer and B is the 4-digit positive integer formed by reversing the digits of A. If A-B is greater than zero and is divisibly by 45. Find the number of possible values of A.

a)248 b) 400 c) 512 d) 805 e) 405

115) Let S denote the set of all (positive) divisors of 60^5. The product of all the numbers in S equals 60^e for some integer e. What is the value of e?

a) 198 b) 99 c) 1980 d) 990 e) none of these

116) Find the number of ordered pair of positive integers $(a.n)$ such that $a^{n+1} - (a+1)^n = 2001$

a) 0 b) 1 c) 2 d) 3 e) none of these

117) How many positive numbers less than 100000 are such that only the digits $5, 6, 7, 8, 9$ are used in the number and the number is divisible by 4?

a) 780 b) 781 c) 313 d) 625 e) 751

118) For all positive integers $n > 1$, $f(n) = \frac{n^2+n}{2} + 2[\frac{f(n-1)-(n-1)}{n-1}]$ *and* $f(1) = 2$. what is the value of $f(9) - f(8)$?.

a) 44 b) 54 c) 11 d) 10 e) none of these

119) The average of three distinct prime numbers between 50 and 75 is $\frac{191}{3}$. Find teh difference between the largest and the smallest prime numbers?

a) 18 b) 12 c) 10 d) 4 e) cannot be determined

120)From one corner of a square field, a boy runs in random direction with random uniform velocity. The greatest distance the boy can run in one minute is the length of the diagonal of the field. What is the probability that the boy will be in the field after the end of one minute?

a) $\frac{1}{\pi}$ b) $\frac{2}{\pi}$ c) $\frac{1}{2\pi}$ d) $\frac{1}{8\pi}$ e) data insufficient

121)If $f(m+1) = m(-1)^{m+1} - 2f(m)$ for integers $m >= 1$, and $f(2001) = 1$, compute $f(1) + f(2) + f(3) + ... + f(2000)$

a) 2001 b) 2000 c) 1001 d) 1000 e) none of these

122) ABC is triangle such that its perimeter is l. Let$m$ *and* n be positive integers such that $l = am = bn$ where a, b *and* c have their usual meanings(sides of the triangle). How many such triangles are isosceles?

a) 0 b) 1 c) 2 d) 3 e) none of these

123)Find $f(10)$,if $f(n) = n - 10$ *when* $n > 100$, *and* $f(n) = f(f(n + 11))$ *otherwise*

a) 91 b) 101 c) 109 d) 99 e) none of these

124) Triangle ABC is right angled at A and has a perimeter of 136. A circle is drawn from A point X on AB such that it is tangent to BC and AC. The radius of the circle is 17 . If BX=$\frac{m}{n}$ where m and n are relatively prime. Then $m + n =$?

a) 20 b) 8 c) 22) d) 85 e) 88

125) Given a point P inside of a quadrilateral ABCD where $< APB =< CPD = 120, AP = BP,$ *and* $CP = DP$. If $K,\ L,\ M$ are midpoints on $AB,\ BC,$ *and* CD respectively. Then

a)$KL = LM$.
b)triangle KLM is equilateral.
c)triangle KLM is isosceles.
d) a and b.
e) a and c.

126) What is the probability that the sum of the digits of a 10-digit number is divisible by 10?

a) 1/5 b) 1/10 c) 1/15 d) 1/20 e) none of these

127) Given the polynomial $F(x) = x^n + a_{n-1}x^{n-1} + ... + a_1x + a_0$ with integral coefficients $a_0, a_1, ..., a_{n-1}$, and given that there exist four distinct integers $a,\ b,\ c,\ d$ such that $F(a) = F(b) = F(c) = F(d) = 5$. Then,

a) there is no integer k such that F(k)=8.
b) there is no integer k such that F(k)=10.
c) there is no integer k such that F(k)=12.
d) there is no integer k such that F(k)=16.
e) all of these

128) $n^4 - 20n^2 + 4 = k$ where k is a prime number and n is an integer. How many such k exist?

a) 0 b) 2 c) 4 d 6 e) none of these

129) Given that A, B and C are angles of a Triangle then $\frac{1}{(cotA+cotB)^2} + \frac{1}{(cotA+cotC)^2} + \frac{1}{(cotC+cotB)^2}$

a) has a maximum of 9/2
b) has a minimum of 9/2
c) has a maximum of 9/4
d) has a minimum of 9/4
e) none of these.

130) An isosceles triangle ABC has vertex angle $x°$ and its sides are $sinx,\ \sqrt{sinx},\ and\ \sqrt{sinx}$. What is its area?

a) 4/25 b) 8/25 c) 16/25 d) 24/25 e) none of these

131) The last 7 digits of $n!$ are 8000000. What is n?

a) 25 b) 26 c) 27 d) 28 e) 29

132) A pythagorean triangle is a triangle with one angle equal to $90°$ and sides of integer length. Find the number of pythagorean triangles whose area is twice the perimeter?

a) 0 b) 1 c) 2 d) 3 e) 4

133) For the triangles found in $problem 132)$, find the sum of areas of all the triangles (in sq units)?
a) 180 b) 300 c) 276 d) 396 e) 450

134) Find all ordered pair of integers (a,b) such that $a(a+1) = b(b+1)(b+2)(b+3)$

a) 8 b) 12 c) 16 d) 24 e) none of these

135) On line $ABCD$, A triangle BEC is drawn such that $m(< EBC) > m(< ECB)$. If $m(< ABE) = 4x+y, m(< E) = 84°$ and $m(< ECB) = x+y$. Then how many integral values of y exist?

a) 19 b) 20 c) 27 d) 28 e) 47

136) Let $a_1 = 1$ and a_n be the number formed by putting the digits of n at the end of a_{n-1}. For $k = 1\ to\ 2008$, compute the number of a_k that are divisible by 3.

a) 2008 b) 1338 c) 669 d) 1004 e) 502

137)Aarav and Vinit have $x\ and\ y$ number of cards. The cards are numbered from $1, 2, 3..x$ and $1, 2, 3...y$ for each of them. Now a pagal comes and asks each of them to do the following operation k number of times where k is $x - 1$ and $y - 1$ for Aarav and Vinit respectively.

Operation: Take any two cards numbered $a,\ b$ (at random) and replace them with a card numbered $ab + a + b$.

Now pagal asks them the final numbers. They just tell him that if Aarav has A and Vinit has V as the final card. Then $A = 6V + 5$. Pagal answers $x + y$ the next moment. What is that ?

a) 7 b) 8 c) 9 d) 10 e) 11

138) In $Quadrilateral\ ABCD\ < B = 90°,\ AB = 3,\ BC = 4\ and\ CD = 5.$

Find the area of the *Quadrilateral*

a) 6 b) 12 c) 18 d) 24 e) none of these

139) Inside a *Rectangle* $ABCD$ there is a point P such that $AP = 6$ $BP = 7$ *and* $CP = 5$ Find DP.

a) $\sqrt{3}$ b)$2\sqrt{3}$ c) $2+\sqrt{3}$ d) $2-\sqrt{3}$ e) none of these

140)Let α be an operation on integers a *and* b such that :

1)$a\ \alpha\ a = a + 2$
2)$a\ \alpha\ b = b\ \alpha\ a$
3)$\frac{a\ \alpha\ (a+b)}{a\ \alpha\ b} = \frac{a+b}{b}$

Find $(21\ \alpha\ 13)$

a) 120 b) 195 c) 840 d) 819 e)1365

141) If $a + b + c = 3$, $a^2 + b^2 + c^2 = 9$, $a^3 + b^3 + c^3 = 24$, Then find $a^4 + b^4 + c^4$

a) 27 b) 69 c) 24 d) 36 e) none of these

142) Given that $f(x + y) = f(x) + f(y) + xy$ for all real numbers x, y. Also $f(4) = 10$ find $f(2008)$

a) 2008 b) 2001 c) 4034072 d) 2017036 e) none of these

143) The positive integers a *and* b are such that the numbers $15a+16b$ *and* $16a-15b$ are both squares of positive integers. Find the least possible value that can be taken by the minimum of these two squares.

a) 13^2 b) 37^2 c) 481^2 d) $(481.13)^2$ e) $(481.37)^2$

144) Given that $p,\ q$ are prime numbers such that $p^2+q = 37q^2+p$. find $p-q$?

a) 3 b) 4 c) 16 d) 36 e) 50

145) Find the number of positive integers that are divisors of at least one of $10^{10}, 15^7, 18^{11}$.

a) 434 b) 435 c) 436 d) 462 d) 461

146) N is a natural number . How many values of N exist, such that $N^2 + 24N + 21$ has exactly three factors?

a)0 b) 1 c) 2 d) 3 e) none of these

147) Determine the number of three digits number N divisibly by 11, and $N/11$ is equal to the sum of squares of the digits of N

a) 0 b) 1 c) 2 d) 3 e) 4

148) Let N be the smallest natural number with the following properties:

a)Its decimal representation has 6 as the last digit
b)If the last digit of N is erased and placed at the leftmost place the new number becomes 4 times the original
if k is the number of digits in N. Find k?

a)3 b) 4 c) 5 d) 6 e)none of these

149)Consider an isosceles triangle and let r be the radius of the circumcircle and ρ be radius of incircle and d be the distance between the centers of the two circles. Then d is :

a) $r(r-2\rho)$ b) $r(r+2\rho)$ c) $\sqrt{r(r-2\rho)}$ d) $\sqrt{r(r+2\rho)}$
e) none of these

150) m *and* n are natural numbers such that $1 \leq m < n$ and 1978^m and 1978^n have the same last three digits. Find $(m+n)$ such that $m+n$ is least.
a) 5 b) 6 c) 7 d) 8 e) none of these

Answer Key

101) c)	102) c)	103)10.lcm(2,3,4..16) +1		104) b)
105) b)	106) c)	107) e)	108) b)	109) c)
110) b)	111) a)	112) d)	113) d)	114) d)
115) c)	116) b)	117) a)	118) d)	119) e)
120) c)	121) e)	122) b)	123) d)	124) e)
125) d)	126) b)	127) e)	128) a)	129) d)
130) b)	131) c)	132) d)	133) d)	134) a)
135) e)	136) b)	137) c)	138) c)	139) b)
140) d)	141) b)	142) c)	143) c)	144) d)
145) b)	146) a)	147) b)	148) d)	149) c)
150) e)				

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